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The use and effectiveness of colorful, contextualized, student-made material for elementary mathematics instruction

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Abstract

Background: There is anecdotal evidence that many elementary teachers integrate mathematics lessons and art activities by having students first make colorful, rich material that is subsequently used in an instructional activity. However, it is unclear whether such activities effectively promote learning and transfer of mathematical concepts. The goal of the present research was to examine the use and effectiveness of such "math-and-art" activities on children's ability to acquire basic fraction knowledge. We report the results of a survey of practicing elementary school teachers in the United States, their use of activities involving physical material, and the resources they use for ideas to supplement the standard curriculum. Two experiments examined first-grade students' learning, transfer, and recognition of fraction knowledge from rich, contextualized material versus simple, generic material.

Results: The survey results confirm that many U.S. teachers use math-and-art activities and are often inspired by informal sources, such as Pinterest and YouTube. Experiment 1 examined the effectiveness of colorful, contextualized student-constructed material (paper pizzas) versus simple, pre-made material (monochromatic paper circles) in an instructional activity on fractions. Students who used the pre-made circles scored higher than those who used the student-made pizzas on pre-instruction tests of basic fraction knowledge, immediate tests of learning, and delayed tests of transfer. Experiment 2 tested students' ability to spontaneously write fractions to describe proportions of pizzas and circles. Students who answered generic circle questions first were markedly more accurate than those who answered pizza questions first.

Conclusions: These findings suggest that rich, contextualized representations, including those made by the student, can hinder students' learning and transfer of mathematical concepts. We are not suggesting that teachers never integrate mathematics and colorful, contextualized material, and activities. We do suggest that elementary students' mathematics learning can benefit when initial instruction involves simple, generic, pre-made material and opportunities for students to make and use colorful, contextualized representations come later.

Keywords: Mathematics learning, Fractions, Instructional activities, Perceptual richness, Contextualization

Introduction

Mathematics is an important part of kindergarten through high school curriculum. In the United States, there are content standards, such as the Common Core Curriculum (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) and specific state standards (e.g., Ohio's Learning Standards for Mathematics, Ohio Department of Education, 2017) that specify exactly what content knowledge students should acquire at each grade level. However, there are no explicit guidelines for how this information is taught. While school systems choose specific textbooks and curriculum, the choices of actual instructional material and activities are often left to the intuition of teachers. Therefore, it is important to examine the effectiveness of material and activities that teachers use, but may not be recommended by formal sources such as official curriculum or educational research journals.

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There is anecdotal evidence that many elementary school teachers incorporate activities, such as art activities, into mathematics lessons perhaps to increase engagement. One particular practice is to have students make representations of mathematical concepts out of everyday material and then use those representations in an instructional activity. For example, students can make colorful geometric quilts, partially filled egg cartons, or proportions of paper pizzas to represent fractions (e.g., Hargrove, n.d.; NCTM, n.d.). A Google search for the keywords "math", "art", "activities" (using a logical "and") yields approximately 156 million results. Including the word "pizza" yields more than 9 million results. These include many vivid images of elaborate projects. The use of such "math-and-art" activities appears to be widespread. However, few studies have explicitly examined the effectiveness of such activities for mathematics instruction. Moreover, we found no strong formal recommendations from groups such as the National Council of Teachers of Mathematics (NCTM) to use such activities. At the same time, there is a plethora of suggestions of ways to incorporate art, crafts, and everyday objects into mathematics instruction from less formal online resources such as social media and blogs.

The goal of the present research was to examine the use and effectiveness of such math-and-art activities on children's ability to acquire basic fraction knowledge. By "math-and-art activity", we mean specifically the incorporation of an art activity into a mathematics lesson by having students first make a representation that will then be used for an instructional activity. To investigate the anecdotal evidence that teachers use math-and-art activities in their classrooms and to better understand what resources they use for ideas, we surveyed practicing elementary school teachers in the United States. In Experiment 1, we tested the effectiveness of material that students made in a math-and-art activity compared to simple pre-made material. Experiment 2 examined the effect of different representations on students' ability to spontaneously label proportions with fractions. First, we discuss evidence that may motivate the use of such activities as well as implications for successful acquisition of mathematical knowledge.

Possible benefits of math-and-art activities

Intuitively, it may seem that incorporating art activities, such as making representations of mathematics from everyday objects, into mathematics lessons may make mathematics seem more fun and hence increase student engagement. In addition, these activities can provide students with physical material to manipulate for instruction and potentially provide a familiar context such as sharing food. There is evidence that both physical material and familiar contexts can be beneficial for mathematics learning.

Instructional activities involving physical manipulatives, such as base-10 blocks and physical number lines, have been shown to be effective in teaching basic number properties (e.g., Fuson, 1986; Fuson & Briars, 1990; Siegler & Ramani, 2009; Tsang, Blair, Bofferding, & Schwartz, 2015; Wearne & Hiebert, 1988). The use of physical manipulatives can benefit the acquisition of other aspects of mathematics. For example, one study demonstrated that children were more accurate solving equivalence problems of the form $a + b = c + \_\_$ when presented with quantities of wooden blocks than when presented with standard mathematical symbols and, in addition, experience solving equivalence problems with these manipulatives led to improvements in solving symbolic equivalence problems (Sherman & Bisanz, 2009; see also Manches & O'Malley, 2016). Such physical instantiations of mathematics may be beneficial for communicating some mathematical concepts because this material often links mathematical properties with perceptual properties of the objects. For example, number magnitudes are correlated with the physical size of base-10 blocks in which the size of a 10 block is ten times that of a unit block and the size of a 100 block is 10 times that of a 10 block. Physical material may also have an advantage for children over strictly symbolic representations by reducing demands on working memory (Zhang & Norman, 1995). For example, for young children, comparing the magnitude of numbers using physical material resembling "||" and "|||" does not require remembering the meaning of symbols, whereas comparing strictly symbolic representations, such as "2" and "3", does. In addition to providing perceptual information and reducing working memory load, the process of interacting with manipulatives may itself promote learning. For example, elementary students who interacted with physical material demonstrated better knowledge of fractions than those who only observed the physical material (Martin & Schwartz, 2005).

While there is evidence that manipulatives can be effective for teaching mathematical concepts, many researchers have pointed out that they are not always effective (e.g. Ball, 1992; Kamii, Lewis, & Kirkland, 2001; Sowell, 1989). Successful learning from physical material requires the learner to recognize that the physical material has a dual nature; it is an object in its own right and also a symbol for the mathematics it is intended to represent (i.e., dual representation, DeLoache, 1995, 2000; Uttal, Schreiber, & DeLoache, 1995). It may be that some physical material is particularly difficult for students to link with the mathematics (Uttal, Scudder, & DeLoache, 1997; Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009).

It is also possible that making or using familiar objects, such as pizzas or candies, in a classroom activity may suggest a familiar context that can facilitate learning (e.g., Mack, 1990). Support for teaching through familiar contexts often stems from theories of situated cognition and situated learning which posit that much of knowledge, including mathematical knowledge, is grounded in real-life experiences and therefore tapping these experiences can promote learning (e.g., Lave & Wenger, 1991; Greeno, 1989; Greeno, Smith, & Moore, 1992). Arguments in favor of situated cognition generally focus on evidence that many abilities, such as mathematical abilities, do not necessarily coexist in formal and informal settings and that people often reason more accurately in familiar settings than in decontextualized abstract contexts (e.g., DeFranco & Curcio, 1997; Greeno, 1989; Guberman, 1996; Lave, 1988; Resnick, 1987; Saxe, 1988). A well-known example is that of Brazilian street children who could perform complex mental arithmetic in street contexts but were unable to solve analogous problems in school contexts (Carraher, Carraher, & Schliemann, 1985; Guberman, 1996; Saxe, 1988). Similarly, children as well as adults often accurately solve problems when presented through a familiar context by using informal strategies. For example, prior to being formally introduced to the concepts of division or fractions, young children can solve problems involving equal sharing of a number of items between a number of people (Frydman & Bryant, 1988; Squire & Bryant, 2002). Additionally, high school and college students are often more successful in solving simple algebra problems when presented as story problems than when presented as symbolic expressions (Koedinger & Nathan, 2004; Koedinger, Alibali, & Nathan, 2008).

Successful acquisition of mathematical knowledge

As discussed, there is evidence that both children and adults are often better problem solvers in familiar real-world domains than in decontextualized abstract domains (although this is not always the case, see Hickendorff, 2013; Koedinger, et al., 2008). However, successful acquisition of mathematical knowledge requires not only an ability to problem solve within a familiar context, but also the ability to apply mathematical knowledge to novel situations including symbolic representations and unfamiliar real-world situations. Integrating mathematics instruction with an activity of making the physical instructional material may hinder acquisition of the mathematical knowledge because the student-made material (e.g., fractions as proportions of paper pizzas) can communicate extraneous information that simple generic material (e.g., fractions as proportions of circles) does not (see Kaminski & Sloutsky, 2011; Kaminski, Sloutsky, Heckler, 2013). This additional information is nonessential to the relations that define the concept. More specifically, such rich student-made material and simple generic pre-made material differ on three dimensions: perceptually rich versus perceptually sparse, contextualized versus decontextualized, and student-made versus pre-made. There are reasons to believe that the added information on each of these dimensions can hinder learning and/or transfer.

Increased perceptual richness adds extraneous superficial information that is typically more salient than to-be-learned mathematical relations. This superficial information may capture the learner's attention, diverting it from the less salient relational information that defines the mathematics. Instantiating simple mathematical concepts with perceptually rich, real objects has been shown to hinder very young children's detection of relations (Mix, 1999; Son, Smith, & Goldstone, 2011). For example, 3- and 4-year-old children more accurately recognized numerical equivalence between two sets of simple, generic objects than between a set of perceptually rich objects and a set of generic objects (Mix, 1999). Instantiating mathematical concepts with perceptually rich, familiar objects may not always hinder initial learning when explicit instruction is given, however, such instantiations can hinder later transfer of mathematical knowledge to novel instantiations. Five-year-olds who were taught to recognize equal proportions between sets of contextualized, perceptually rich objects (i.e., images of colorful cupcakes) were less able to recognize equal proportions between sets of novel objects than 5-year-olds who learned this concept with decontextualized, perceptually sparse objects (i.e., images of monochromatic circles) (Kaminski & Sloutsky, 2009). The inclusion of extraneous perceptual information can also hinder children's ability to acquire new mathematical procedures; kindergarten and first-grade children were less able to learn to read bar graphs when the bars contained images of colorful, familiar objects than when the bars were monochromatic (Kaminski & Sloutsky, 2013). In some cases, realistic, perceptually rich material can hinder mathematical problem solving; elementary school children were less accurate solving word problems involving money when they were given bills and coins to help them solve the problems than children who were not given the physical material (McNeil, Uttal, Jarvin, & Sternberg, 2009). In addition, increased perceptual richness of symbols has been shown to hinder both the learning and transfer of a novel algebraic concept for undergraduate students (Sloutsky, Kaminski, & Heckler, 2005).

As discussed previously, familiar contexts can often facilitate problem-solving, but this is not always the case. Sixth-graders were less accurate on multi-digit division problems when presented in a contextualized format than when presented in a strictly numerical format (Hickendorff, 2013). Similarly, undergraduate students were less accurate solving complex (as opposed to simple) algebra story problems than analogous symbolic equations (Koedinger, et al., 2008). It has also been shown that some familiar contexts that facilitated undergraduate students' initial learning of a mathematical concept hindered their subsequent transfer (Kaminski, Sloutsky, & Heckler, 2008, 2013).

Finally, it is possible that the act of constructing the material prior to instruction may hinder learning or transfer. Successful acquisition of mathematics from physical manipulatives may mean that the learner views the material not just as an object itself but as a symbol for the mathematics it is intended to represent (see dual representation, DeLoache, 1995, 2000; Uttal, et al., 1995). Allowing preschool children to play with a scale model of a room for 10 minuntes markedly decreased their ability to use the model as a symbol for a real room (DeLoache, 2000). Playing with the model increased children's non-symbolic use of the material, which likely encouraged them to see the material more as an object itself and not as a symbol for something else (see DeLoache, 2000; Uttal, 2003). The act of children making colorful, contextualized material as an art project is also non-symbolic with respect to the relevant mathematics. Therefore, the act itself may hinder acquisition of the mathematics.

Successfully acquiring mathematical knowledge from perceptually rich, contextualized instantiations may require sufficient ability to inhibit irrelevant information and focus attention on the underlying relations that define the mathematics. For young children, filtering of irrelevant, potentially distracting information is particularly difficult (Kemler, 1982; Shepp & Swartz, 1976; Smith & Kemler, 1978, see also Hanania & Smith, 2010), but improves substantially with age (Davidson, Amso, Anderson, & Diamond, 2006). For example, when 6- and 9-year-olds were instructed to sort items according to shape, with color being an irrelevant dimension, 6-year-olds, but not 9-year-olds, were slower when color varied independently of shape than when color co-varied with shape or did not vary at all (Shepp & Swartz, 1976).

Given these developmental changes, it is possible that elementary school children have sufficiently developed inhibitory and attentional control to learn mathematics from rich, contextualized material such as that made in an art activity. Moreover, such material may be more interesting and engaging than bland, decontextualized material and as a result have an advantage over generic material. On the other hand, while the negative effects of perceptual richness and contextualization of simple concepts such as numerical equivalence, shape categorization, and simple spatial relations attenuate with development (see also DeLoache, 1995, 2000), perceptually rich, contextualized representations of more advanced mathematical concepts can hinder even adults' ability to acquire mathematical knowledge (Kaminski, et al. 2008, 2013, Sloutsky, et al., 2005; see Goldstone & Sakamoto, 2003 and Son & Goldstone, 2009 for similar findings). Adults may fail to recognize common relations between advanced mathematical systems because they are much more complex than simple mathematical concepts, such as those based on cardinality. Increased relational complexity is known to hinder relational reasoning (Andrews & Halford, 2002; Halford, 1993; Halford, Andrews, Dalton, Boag & Zielinski, 2002). For example, children are less likely to reason relationally about ternary relations than about less complex binary relations (Richland, Morrison, & Holyoak, 2006). Therefore, while children's ability to filter irrelevant information improves by elementary school, they are acquiring increasingly complex mathematical concepts such as multiplication, division, and fractions. Reasoning about new and complex mathematical relations instantiated with perceptually rich, contextualized, student-made material, as opposed to perceptually sparse, decontextualized, pre-made material, may make it more difficult to inhibit the superficial information and attend to the less salient mathematical relations.

The present study

The present research examined the use and effectiveness of math-and-art activities in which students make representations of mathematics that are subsequently used in an instructional task. First, we present the results of a survey of practicing teachers in the United States and their use of activities for classroom instruction, their beliefs in the effectiveness of various activities, and the resources they use for ideas to supplement the standard curriculum. While many types of instruction involve activity, we are using the term "activity" to refer to those that involve students using physical material and manipulating the material to represent mathematics. The goal of Experiment 1 was to examine the effectiveness of a math-and-art activity for teaching basic fraction knowledge. Such an activity is effective only to the extent that the material made by the students effectively promotes initial learning and subsequent transfer of the relevant mathematics. Therefore, we examined the effectiveness of the material by comparing learning and transfer from student-made representations of fractions (proportions of paper pizzas) versus simple pre-made representations of fractions (sectors of paper circles). The goal of Experiment 2 was to examine how the visual representations themselves, without constructing them, affect students' spontaneous ability to label proportions with fractions.

We chose to examine children's acquisition of fraction knowledge for two reasons. First, the concept of fractions is a prerequisite for other mathematical concepts, including probability, proportional reasoning, algebra, and many higher-order concepts. There is evidence that knowledge of fractions in elementary school is predictive of acquisition of algebraic knowledge in middle school and general mathematics achievement in high school (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; Siegler, Duncan, Davis-Kean, Duckworth, Claessens, Engel, Susperreguy, & Chen, 2012). Therefore, successful acquisition of fraction knowledge is crucial for future success in mathematics. Second, young children enter school with informal knowledge of equal sharing and proportions; they are typically successful on simple proportional reasoning tasks involving familiar real-world contexts (Mack, 2001; Mix, Levine, & Huttenlocher, 1999; Singer-Freeman & Goswami, 2001; Spinillo & Bryant, 1991, 1999). However, difficulties arise for older children, with fraction tasks involving standard mathematical notation. Children often make errors on magnitude comparisons, estimations, and arithmetic (Hiebert, 1992; Hiebert & Wearne, 1986; Kieran, 1992; Kouba, Carpenter, & Swafford, 1989; Kouba, Zawojewski, & Strutchens, 1997). For example, when judging which of two fractions is larger in magnitude, children frequently base their responses on the largest integer present (e.g., Alibali & Sidney, 2015; Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004). If shown 1/2 and 1/3, children often respond that 1/3 is larger because 3 is larger than 2.

Given that the perceptual richness, contextualization, and non-symbolic use of material each can communicate extraneous information, that fractions are a difficult concept for elementary school children, and that attentional focus is not fully mature at this age, we hypothesized that material made by students in a math-and-art activity will be less effective for initial learning and subsequent transfer of basic fraction knowledge than simple, pre-made material. Experiment 1 tested this hypothesis. Children who had recently completed first grade participated in an instructional activity on basic fractions. Half of the participants made their material out of construction paper and the other half used simple, pre-made material. Another goal of this experiment was to examine student learning and transfer in a controlled experimental situation that is similar to a classroom setting. To do so, our participants attended a two-day "learning camp" for 3.5 hours per day where they engaged in different activities. The activities were primarily mathematical with group instruction.

In addition to predicting that the material made in the math-and-art activity will hinder explicit learning and transfer of mathematics, we also expected that the perceptually rich, contextualized representations themselves (in absence of making them) can divert attention from previously learned mathematical relations. Therefore, we hypothesized that perceptually rich, contextualized representations will hinder children's spontaneous ability to recognize previously acquired fraction knowledge in comparison to generic, decontextualized material. Experiment 2 tested this hypothesis. Children who had prior instruction on basic fractions in their regular classrooms were shown perceptually rich, contextualized representations or generic representations (i.e., proportions of pizzas and circles, similar to those used in Experiment 1) and asked to write the fractions that describe the proportions shown.

Survey

Method

Participants

Participants were 413 practicing elementary school teachers in the United States who teach mathematics in any grade from kindergarten through grade 4. To recruit participants, a list of 1400 randomly selected public and private schools in the United States was generated from school lists retrieved from the National Center for Education Statistics, U.S. Department of Education (downloaded from https://nces.ed.gov/programs/edge). The list was narrowed to a total of 691 schools (561 public,130 private) to include only elementary schools for which teachers' email addresses were available on the school website. A total of 11,666 teachers were sent email messages. Participants were teachers who responded, indicated that they teach mathematics, and completed at least 80% of the survey.

Materials

The survey consisted of 18 questions. Questions 1 and 2 asked participants what type of resources they have used, would consider using, or would not use to obtain ideas for instructional activities. Question 1 presented a list of various categories of information resources including hardcopy journals, professional meetings, and online resources of any type (see Table 1). Question 2 presented a list of specific types of online resources (see Table 1). Questions 3–6 assessed participants' beliefs about student engagement and their use of activities in math lessons. Specifically, they were asked to indicate the extent to which they agree with the following statements on a scale from 1 (strongly disagree) to 5 (strongly agree).

3. It is difficult to keep children engaged in mathematics lessons.
4. Incorporating activities into math lessons can increase students' engagement in math learning.
5. Incorporating activities into math lessons can increase students' math learning outcomes.
6. On a scale from 1 (not at all) to 5 (extremely), how challenging do you feel it is to promote children's interest in learning mathematics?

Table 1 Percentage of teachers responding to their use of different types of resources

	Have used	Have not used, but would consider using	Would not use
Discussions with colleagues	98%	2%	–
Online resources	96%	4%	*
Professional development meetings	92%	8%	*
Retail stores	81%	15%	4%
Professional conferences	75%	24%	1%
Hard copy journals	47%	40%	13%
Specific online resources			
Teachers Pay Teachers	95%	4%	1%
Pinterest	85%	10%	5%
YouTube	80%	15%	5%
Blogs	51%	35%	14%
Facebook	40%	25%	35%
Online journals NCTM	37%	57%	6%
Online education journals not NCTM	32%	57%	11%
Instagram	25%	35%	40%
Twitter	16%	36%	48%
Reddit	3%	50%	47%
Periscope	2%	58%	40%

Note: Percentages are rounded to the nearest ones place
*denotes ≤ 0.5%

Questions 7 and 8 asked participants how often they incorporate activities into their mathematics lessons and when using activities, what percent of time are they inspired by material or resources other than the school-designated curriculum.

Each of the questions, 9–18 presented a description of a particular activity involving fractions along with a photograph of the associated representation of fractions (see Table 2). The representations included an area model (i.e., circle), number line, and a tape diagram (i.e., paper strips); these are the only visual fraction models that are explicitly described in the Common Core Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The other seven representations involved everyday objects or student-constructed material, such as paper pizzas, kites, and quilts. Note that the circle and pizza activities are short descriptions of the activities used in Experiment 1. Participants were asked if they have used a similar activity or would consider using the activity in the future. They were also asked to rate the effectiveness of the activity. Participants were told, "Each description is of an activity that represents fractions in a particular manner. Specific instruction on fractions would follow and/or be incorporated into the activity. Note: if fractions are not part of the grade level you teach, please respond as though you were teaching that grade level and indicate what you would think is appropriate".

Finally, the respondents were asked whether they teach mathematics in their classrooms, the number of years they have been teaching, the state and type of community (urban, suburban, small town, or rural) in which their school is located.

Procedure

Teachers were contacted via email and asked to complete an anonymous online survey about their use of activities to supplement mathematics lessons and what resources they use for ideas for such activities. The survey was administered via Qualtrics (Qualtrics, LLC, Provo, Utah, USA). Questions 9–18 were presented in random order.

Results

The mean portion of the survey completed by participants was 99% (*SD* = 2%). The mean number of years teaching was 14.9 (*SD* = 9.7). As indicated by teacher responses, 120 participants were from the Midwest, 59 were from the Northeast, 149 were from the South, and 85 were from the West. Eighty-eight respondents were from urban areas; 153 were from suburban areas; 102 were from small towns; 70 were from rural areas. Fifty-four were from private schools; 359 were from public schools. Seventy participants were kindergarten teachers; 191 taught first or second grade; 152 taught third or fourth grade. Except for the exceptions noted below,

Table 2 Teacher use and mean ratings of the effectiveness of various fraction activities

Activity	Description	Percentage of respondents		Mean rating of effectiveness (1: not at all–5: extremely)
		Have used	Would use in the future	
Candies	Have students bring in two chocolate bars. Identify fractions, such as 1/2, 1/4, 1/8, as proportions of the candy bars.	60%	90%	3.95 (.84)
Circles	Have students use circles cut into equally sized sectors to represent fractions such as 1/2, 1/4, 1/8.	78%	96%	3.74 (.86)
Cookies	Have students make sets of cookies (either real or paper). Construct sets of cookies with and without chocolate chips to represent fractions such as 1/2, 1/4, 1/8.	56%	89%	3.96 (.88)
Egg Carton	Have students bring in empty egg cartons. Fill egg cartons with different numbers of plastic eggs to represent fractions such as 1/2, 1/4, 1/12.	21%	84%	3.68 (.90)
Kite	Have students make a kite of different colored sections to represent a fraction. Represent common fractions, such as 1/2, 1/4, 1/8, through kites made by different students.	33%	84%	3.57 (.89)
Number Line	Have students make a number line to represent fractions such as 1/2, 1/4, 1/8.	70%	87%	3.30 (.99)
Paper strips	Have students cut strips of paper. Fold the strips into different proportions to represent fractions such as 1/2, 1/4, 1/8.	74%	95%	3.85 (.85)
Pattern blocks	Have students explore a set of pattern blocks and identify fractions, such as 1/2, 1/4, 1/8, as the ratio of a smaller shape to a larger shape.	70%	94%	3.78 (.88)
Pizzas	Have students make pizzas out of paper. Cut the pizzas into equally sized slices to represent fractions such as 1/2, 1/4, 1/8.	67%	93%	3.86 (.82)
Quilts	Have students make paper quilts by coloring (or pasting) equal-sized geometric parts of a square. Identify the fraction (such as 1/2, 1/4, 1/8) which describes the proportion of each color or pattern in a specific square.	23%	85%	3.61 (.90)

Note: Standard deviations appear in parentheses

differences in responses across geographic region of the United States (Midwest, Northeastern, South, West), community type (urban, suburban, small town, rural), public or private school, and grade level were either insignificant, *p*s > .10, or small (i.e., differences < 4% or effect size $\eta_p^2 < 0.04$). Results were combined for all participants across geographic region, community type, school type, and grade level.

Overall, the respondents neither strongly agreed nor disagreed that it is challenging to promote children's interest in learning mathematics ($M = 2.61$, $SD = 0.92$ on a scale from 1, strongly disagree, to 5, strongly agree). Teachers neither strongly agreed nor disagreed that it is difficult to keep children engaged in a mathematics lesson ($M = 2.73$, $SD = 1.01$). Teachers did strongly agreed that incorporating activities into math lessons can increase students' engagement in math learning ($M = 4.52$, $SD = 0.93$) and increase students' math learning outcomes ($M = 4.44$, $SD = 0.94$). With respect to the frequency of activities, 78% of respondents (93% kindergarten, 73% grades 1–2, 77% grades 3–4) indicated that they incorporate activities into math lessons one or more times per week; 17% (4% kindergarten, 22% grades 1–2, 16% grades 3–4) indicated that they do so one to four times per month; 3% said they use activities five to ten times per year; 2% said they use activities one to five times per year. Less than 1% indicated never incorporating activities into mathematics lessons. The difference in frequency of activity use across grade levels was significant, χ^2 (8, $N = 413$) > 17.7, $p < 0.03$. There was a small positive correlation between frequency of activity use and the belief that activities increase learning outcomes, $r = 0.12$, $p < 0.02$, $n = 413$. There were small negative correlations between frequency of activity use and the belief that it is challenging to promote students' interest in mathematics learning, $r = -0.10$, $p < 0.05$, $n = 413$, and the belief that it is difficult to keep children engaged in mathematics lessons, $r = -0.21$, $p < .001$, $n = 412$. When incorporating activities into math lessons, teachers responded that their choices of activities are inspired by material or resources other than the school-designated curriculum on average 68% of the time ($M = 68.2\%$, $SD = 26.0\%$).

Table 1 presents the data on the use of specific resources and the percentage of respondents who indicated that they have used, would use, or would not use them. All of the teachers responded that they use resources other than the designated curriculum. Ninety-six percent indicated that they use online resources. Surprisingly, a minority of the teachers said they use formal online educational resources (i.e., 37% NCTM material, 32% non-NCTM educational material). In contrast, 85% said they use Pinterest and 80% said they use YouTube. The most popular online resource, used by 95% of respondents, was Teachers Pay Teachers, which is a website that provides free and paid material created by teachers that other teachers can download.[1]

Table 2 presents the percentage of teachers who responded that they have used or would use particular activities and their mean ratings of the effectiveness of each. The majority of respondents reported having used candies, circles, cookies, paper strips, pattern blocks, pizzas, and the number line. The most commonly used representation was circles, which was used by more teachers than all the other representations (McNemara χ^2s > 6.89, *p*s < 0.05, adjusted for multiple comparisons) except paper strips, McNemara χ^2 (1, N = 410) = 2.39, p = 0.12. The survey results show that teachers tend to use number lines and paper strips more than most of the representations involving real-world objects (McNemara χ^2s > 12.2, *p*s < 0.01 compared to the use of candies, cookies, egg cartons, kites, quilts, and adjusted for multiple comparisons).

However, the percentage of teachers who have used pizzas was equivalent to the percentages who have used the number line (McNemara χ^2 (1, N = 410) = 1.01, p = 0.32). Teachers also reported that they are more likely to use pizzas in the future than to use the number line (McNemara χ^2 (1, N = 409) = 6.96, p < 0.01) and equally likely to use candies, cookies, egg cartons, kites, and quilts in the future as the number line (McNemara χ^2s < 1.74, *p*s > 0.18). With respect to the activities used in Experiment 1, 96% of teachers responded that they would use the activity involving circles (the generic activity), and 93% responded that they would use the activity involving student-made paper pizza (the math-and-art activity).

On average, participants rated all of the activities as effective. With respect to the activities of Experiment 1, respondents rated the pizza activity as somewhat more effective than the circle activity, paired sample $t(407)$ = 2.86, p < 0.01. Teachers also rated the number line as less effective than all of the other representations, paired sample *t*s > 4.84, *p*s < 0.001.

The results of the survey support anecdotal evidence that many U.S. teachers regularly use activities in mathematics instruction, including activities that involve students making representations of fractions out of paper and everyday objects. Teachers responded that they believe these activities effectively promote learning. The results also reveal that teachers use many informal, non-research-based resources for inspiration for such activities. The goal of Experiment 1 was to test the effectiveness of a math-and-art activity for fraction instruction by examining learning and transfer from rich, student-made material versus simple, pre-made material.

Experiment 1

Participants were taught basic fractions knowledge. Because prior research has shown that children are more accurate solving proportion problems involving continuous representations (e.g., proportions of circles or pizzas) as opposed to discrete representations (e.g., number of dots or candies) (Cramer & Wyberg, 2009; Singer-Freeman & Goswami, 2001; Spinillo & Bryant, 1991, 1999; see also Mix, et al., 1999), we used continuous representations for instruction. Two between-subjects conditions (student-made art and pre-made generic) varied the instructional material. In the student-made art condition, participants first made pizzas from construction paper. They then used their pizzas for the instructional activity. In the pre-made generic condition, participants made unrelated paper pictures and then were given monochromatic paper circles for the instructional activity. Participants were then tested on their ability to label proportions of novel objects with fractions, label estimated proportions of continuous quantities with fractions, and compare fraction magnitudes.

Method

Participants

Twenty-nine children who just completed first grade were recruited from public schools in suburbs of a Midwestern city in the United States (18 girls and 11 boys, M = 7.31 years, SD = 0.35 year). The majority of participants were Caucasian from middle-class families. Parents gave written consent for their children to participate in a mathematics learning research program. Two separate sessions (i.e., dates of participation) were held during early summer in two classrooms of a building located off the university campus. Ten children participated in the first session; 19 children participated in the second session. Within each session, participants were randomly assigned to experimental conditions.

Design

Participation involved attendance on two 3.5-hour morning meetings (subsequently referred to as day 1 and day 2). On day 1, participants were given instruction on fractions. On day 2, which occurred one week after

[1]There were some notable differences across groups in the proportion of teachers who indicated that they would not use Facebook, Instagram, and Twitter. Specifically, more teachers in the West reported being unwilling to use these resources than elsewhere, χ^2s > 4.78, *p*s < 0.03 (Facebook: 32% Midwest, 36% Northeast, 30% South, 47% West; Instagram: 35% Midwest, 45% Northeast, 34% South, 54% West; Twitter: 48% Midwest, 47% Northeast, 43% South, 59% West). More teachers in the rural communities reported being unwilling to use Instagram and Twitter than elsewhere, χ^2s > 5.65, *p*s < 0.02 (Instagram: 35% urban, 35% suburban, 45% small town, 51% rural; Twitter 40% urban, 43% suburban, 56% small town, 60% rural). Also, more private school teachers reported being unwilling to use Twitter than public school teachers (63% and 46% respectively), χ^2 (1, N = 397) > 4.84, p < 0.03.

day 1, they were tested on the topics taught on day 1. Participants were randomly assigned to one of two between-subject conditions, student-made art (N = 15) or pre-made generic (N = 14), which varied the material they used on day 1. Testing on day 2 was identical across the two conditions. All instruction, activities, and testing were presented by an experimenter (i.e., instructor) to groups of participants.

Materials

The to-be-learned aspects of fraction knowledge were basic fraction labeling (i.e., label a proportion in a visual display with a fraction) and fraction magnitude. Examples of fraction labeling are to label the proportion of pizza remaining in Fig. 1 and the proportion of the circle that is shaded in Fig. 1 as 1/4. The presented fractions had numerators no greater than 10. Participants were not taught or tested on equivalent fractions or fractions larger than 1.

Day 1 activities consisted of a pre-instruction test, an art activity, fraction instruction, a break, and an unrelated mathematics activity. Day 2 consisted of a fraction test and unrelated mathematics activities with an intermittent break. Table 3 presents an outline of the content and format of assessments given on days 1 and 2.

Day 1: Pre-instruction test The pre-instruction test was a paper and pen test consisting of five open-ended fraction-labeling questions (proportions of pizza in the student-made art condition, proportions of circles in the pre-made generic condition) and six binary-choice magnitude comparison questions. In each condition, the pizzas or circles were divided into equally sized slices with black demarcation lines (as shown in Fig. 1). The magnitude comparison questions were in the same format for both conditions; participants were asked to circle the larger of two fractions. For three of six questions, the correct answer was the fraction with the smallest integer present (e.g., 1/2 and 1/3). The length and extent of the pre-instruction test were short in comparison to that of the tests given during and after instruction because there is evidence that act of answering pre-test questions can negatively affect transfer (Opfer & Thompson, 2008). We wanted to minimize any negative effects of pretesting by administering a minimal pre-instruction test.

Day 1: Pre-instruction art activity The art activity lasted 30 minutes. In both conditions, participants made three separate items from pre-cut colored construction paper. In the student-made art condition, participants assembled and glued pre-cut pieces of paper resembling pizza crusts, sauce, cheese, and toppings. In the pre-made generic condition, participants assembled and glued pre-cut geometric shapes onto rectangular paper.

Day 1: Fraction instruction and testing A 45-minute fraction instructional period followed the art activity. The instructional period began with explicit instruction on fractions, followed by an activity using paper pizzas or circles, depending on the condition. Participants were given booklets to record their answers to questions.

Explicit instruction presented by the instructor included four examples of fractions representing proportions of pizzas in the student-made art condition and circles in the pre-made generic condition. These examples were presented on 8.5-in. × 11-in. sheets of paper; the pizzas/circles were 8 in. in diameter and were divided into equally sized slices (as shown in Fig. 1). Afterward, participants were given four multiple-choice fraction-labeling questions. The response choices included: (1) the correct response, (2) correct numerator, but incorrect denominator, (3) correct denominator, but incorrect numerator, and (4) incorrect numerator and incorrect denominator. The order of the answer choices was counterbalanced across questions.

The instructional activity involved the use of the participants' pizzas (in the student-made art condition) or circles (in the pre-made generic condition). In each condition, one representation (pizza or circle) was cut into

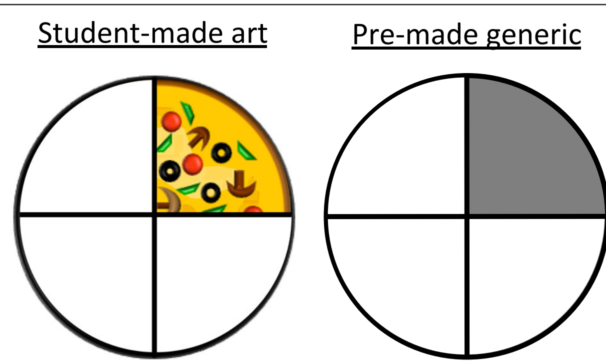

Fig. 1 Example of material in the student-made art and pre-made generic conditions of Experiment 1

Table 3 Format of assessments given in Experiment 1, split by mathematical content and the day of participation

	Day 1		Day 2
Content	Pre-instruction test	Test during instruction	Delayed test
Labeling	Condition-specific	Condition-specific	Novel contextualized
Magnitude Comparison	Numerals only	Condition-specific	Numerals only and contextualized
"How many"	–	Condition-specific	–
Estimation	–	–	Contextualized and generic

Note: Condition-specific refers to pizzas in the student-made art condition and circles in the pre-made generic condition. Unless denoted condition-specific, the format of questions was identical across condition

two halves. Another was cut into four fourths. The third was cut into eight eighths. Participants were given 14 fraction stickers (two 1/2, four 1/4, and eight 1/8) to first label proportions of their pizzas or circles. Afterward, they answered six "how many" questions. The "how many" questions asked the number of smaller fractions that would be needed to comprise a larger fraction or whole. Recognizing that a fraction a/b, where $b \neq 0$, is equal to the sum of fractions of the form $1/b$ is part of the standard curriculum (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). More details of the instructional activity appear in the procedure section. Participants were then given six fraction magnitude comparison questions in which they were asked to circle the larger of two fractions. These were the same six questions asked in the pre-instruction test.

Day 2: Delayed test The second day of participation occurred one week after the first day. Participants were given a 65-question multiple-choice test (16 fraction labeling questions, 13 estimation questions, 32 magnitude comparison questions, 4 contextualized magnitude comparison questions). There were four possible response choices for the labeling and estimation questions, and two possible response choices for the magnitude comparison questions.

The fraction labeling and estimation questions were projected onto a screen on the wall using a projector. Participants had paper booklets with the response choices for each question, and they circled their responses in the booklet. Eight of the fraction labeling questions presented a proportion of an object or collection of objects and participants needed to choose the fraction that correctly described the proportion (see left panel of Fig. 2 for an example). Another eight labeling questions presented a fraction and participants needed to choose a picture with a proportion that matched the fraction (see the right panel of Fig. 2 for an example). For both types of questions, there were four types of responses: correct answer, correct numerator/incorrect denominator, correct denominator/incorrect numerator, and incorrect numerator/incorrect denominator. The order was counterbalanced across questions.

The estimation questions presented proportions without demarcation lines between the sections; participants needed to choose the fraction that approximated the proportion shown. Five estimation questions presented proportions of pizza. Five questions presented proportions of circles that were blue (see left panel of Fig. 3 for an example). The five pizza questions were isomorphic to the five circle questions. Three additional estimation problems presented partially filled containers (see right panel of Fig. 3 for an example).

The 32 numerical magnitude comparison questions were presented on paper. Participants were asked to circle the larger of two fractions. The contextualized magnitude comparison questions were shown through the projector and presented short story problems in which two children had proportions of objects. Participants were asked to choose the child who had more and circled their responses in the booklets.

Procedure

The student-made art and pre-made generic conditions were conducted in separate rooms. Children sat in a well-spaced arrangement at small tables. Instruction was given to the entire group of participants, but children completed the mathematical activities and tests individually. A second experimenter was present and acted as an assistant. During the activities, corrective feedback was given by both experimenters to individual children. All fraction instruction and feedback were scripted and completely analogous across the two conditions. The scripts appeared on either the top or the back of the instructional material to remind or prompt the instructor of the precise language to use.

On day 1, participants first answered the pre-instruction test questions and then began the art activity. In the student-made art condition, participants made three paper pizzas. Afterward, the experimenter cut them each into equally sized slices and then returned them to the participant. In the pre-made generic condition, participants were shown examples of

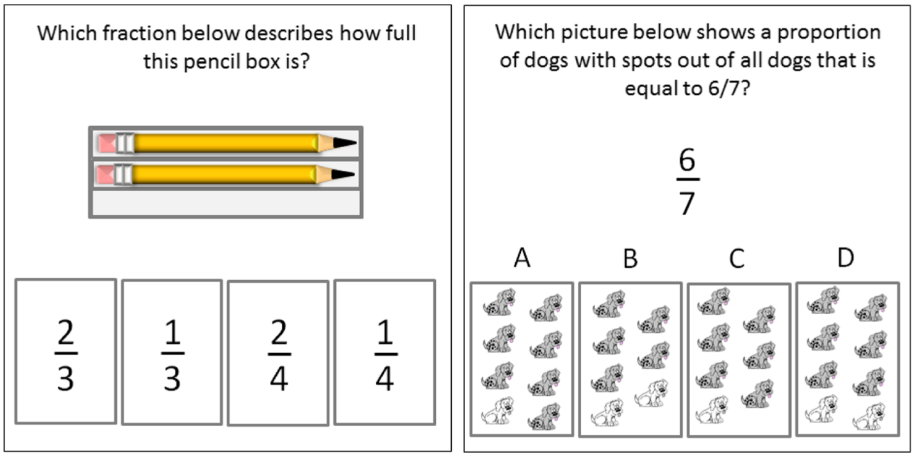

Fig. 2 Examples of fraction labeling questions from the delayed test of Experiment 1

abstract paintings and then made three different abstract pictures that resembled either real objects or an arbitrary design.

The art activity was followed by the fraction instructional period that began with explicit instruction on fractions and then an activity. The instructor first showed participants four examples of fractions representing proportions and then presented the multiple-choice questions. Participants circled their responses in their answer booklet. For each question, after all participants circled their responses, corrective feedback was given to the group by the instructor and then to each individual child by either the instructor or the assistant. Feedback indicated the correct response and explained the relation between the fraction and the proportion. For example, in the pre-made generic condition, the instructor or assistant showed that 1/4 describes the proportion showing one blue part out of four parts altogether, gesturing and counting each part.

Next, participants were asked to use their pizzas/circles to represent different fractions. First, they were shown the fraction 1/4 and asked to consider the pizza/circle that was cut into four pieces and to remove some of the pieces to show a proportion that is 1/4. The instructor then demonstrated this with separate physical material and explained that each single piece can represent 1/4. Participants then placed stickers showing 1/4 on each of the four pieces. Assistance was given to each participant individually to ensure that participants labeled the proportions correctly. Participants did the analogous task for 1/2 and 1/8 and received assistance.

Afterward, participants answered six “how many” questions by using their pizzas/circles. The first of these questions was “how many 1/4s make a whole pizza/

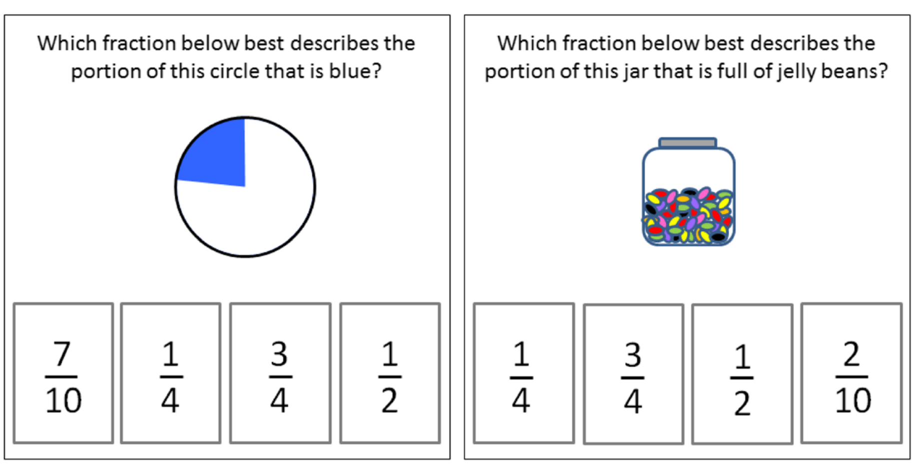

Fig. 3 Examples of estimation questions from the delayed test of Experiment 1

circle?" The instructor demonstrated with separate material that four 1/4 pieces would constitute the entire pizza/circle. Participants answered the remaining five questions, which involved 1/8, 1/4, and 1/2 (e.g., "how many 1/8s make 1/2 of a circle?"), and wrote their responses in their booklets. Participants were then given the six fraction magnitude comparison questions. Participants could use their fraction representations to help them answer. For each question, after all participants had indicated an answer, corrective feedback was given by the instructor to the group and subsequently to each individual child. Feedback indicated the correct response and explained the relation between the fraction and the proportion. Upon completion of the instructional period, children were given a 25-minute break, followed by unrelated mathematical tasks.

On day 2, children were randomly divided into the two classrooms for testing. Participants were first given paper magnitude comparison questions, followed by labeling questions, estimation questions, and contextualized magnitude comparison questions that were projected onto a large wall screen. The experimenter read the questions one at a time. The response choices appeared on the screen and also in the paper booklets. Participants circled their responses in the booklets. After completion of the testing, children had an extended break, followed by unrelated activities.

Results

One child in the pre-made generic condition was not present on the second day and therefore was excluded from the analysis. An additional child in the pre-made generic condition was also excluded because the day 2 score on fraction labeling was more than three standard deviations from the mean. This child scored only 25% correct on day 2 labeling questions, while the mean of the group was 93% and the next lowest score in the group was 81%.

Table 4 presents accuracy on assessments, split by instructional condition and question content. Scores during instruction reflect participants' initial responses prior to corrective feedback. The following sections present the analysis of these results. We analyzed the effect of instructional condition on total scores during instruction on day 1 as a measure of initial learning and total scores on Day 2 as a measure of transfer. Subsequently, we considered the effect of condition on each type of question. Pre-instruction labeling scores and pre-instruction magnitude comparison scores were analyzed separately since labeling questions were condition-specific while the magnitude comparison questions were the same in both conditions.

Day 1: Pre-instruction test

There were no differences between conditions in pre-instruction scores on the magnitude comparison questions, independent-samples t-test, $t(25) = 0.606$, $p = 0.55$. Mean score in the student-made art condition was 56.7% (SD = 15.2); mean score in the pre-made generic condition was 61.1% (SD = 22.8). In both conditions, mean scores were not reliably above a chance score of 50%, one-sample, ts < 1.71, ps > 0.11. A score of 50% would also occur if a participant chose the fraction that had the largest integer. In the student-made art condition, 53% of participants responded by choosing the fraction with the largest integer; in the pre-made generic condition, 58% of participants did so.

While there were no significant differences between conditions on the magnitude comparison questions, there were differences on the labeling questions. These questions were condition-specific. In the student-made art condition, proportions of pizza remaining were shown, and in the pre-made generic condition, proportions of circles shaded grey were shown. Participants in the pre-made generic condition scored higher than those in the student-made art condition (M = 56.7, SD = 40.8 in the pre-made generic condition and M = 22.7, SD = 34.5 in the student-made art condition), independent-samples t test, $t(25) = 2.34$, $p < 0.03$, Cohen's $d = 0.90$. There are two possible explanations for this result. First, the difference may have stemmed from the format of the questions. The simple generic format may be more likely to activate prior fraction knowledge than the contextualized pizza format, leading to higher accuracy in the pre-made generic condition (a possibility that we explicitly tested in Experiment 2). Second, the difference may be due to sampling, resulting in differences in participants between the conditions. Although participants were randomly assigned to conditions, it is possible that participants in the pre-made generic condition happened to have greater prior knowledge of fractions than those in the student-made art condition which led to higher scores on the pre-instruction labeling questions.

The following data analysis assumed that participants in both conditions had equivalent prior knowledge levels and that the differences in pre-instruction labeling were the result of the question format. Experiment 2 was a direct test of the hypothesis that the different question formats are responsible for different levels of accuracy on spontaneous fraction labeling.

Day 1: Test during fraction instruction

The total scores during instruction were markedly different between the two instructional conditions (see Table 4). The combined scores on labeling, magnitude comparison, and "how many" questions were submitted to an analysis of covariance with instructional condition as a fixed factor and pre-instruction magnitude comparison as a covariate. Participants in the pre-made generic condition scored significantly higher than those in the student-made art condition, $F(1, 24) = 9.51$, $p < 0.01$, $\eta_p^2 = 0.28$. There was

Table 4 Mean accuracy (percent correct) on assessments in Experiment 1, split by condition and question content

Question Content	Test during instruction		Delayed test	
	Student-made art (n = 15)	Pre-made generic (n = 12)	Student-made art (n = 15)	Pre-made generic (n = 12)
Labeling	83.3 (20.4)	87.5 (16.9)	69.2 (25.1)	92.7 (6.4)[b]
Magnitude Comparison	65.6 (20.4)	83.3 (18.8)[a]	57.6 (15.9)	68.1 (20.9)
"How many"	56.7 (31.4)	84.7 (18.1)[a]	–	–
Estimation	–	–	54.9 (26.3)	69.9 (24.6)
Total Score	66.7 (14.7)	84.9 (14.5)[b]	59.9 (14.7)	74.4 (16.9)[a]

Note: Standard deviations appear in parentheses.
[a]Significant differences between conditions at $p < 0.05$
[b]Significant differences between conditions at $p < 0.01$

no effect of pre-instruction magnitude comparison score, $F(1, 24) = 1.18$, $p = 0.29$. Also note that although pre-instruction labeling scores differed across condition, an analysis of covariance with both pre-instruction labeling scores and magnitude comparison scores also reveals a significant effect of condition on the combined learning score, $F(1, 23) = 5.17$, $p < 0.04$, $\eta_p^2 = 0.18$, with no effect of pre-instruction labeling score, $F(1, 23) = 1.29$, $p = 0.27$, and with no effect of pre-instruction magnitude comparison score, $F(1, 23) = 0.03$ $p = 0.87$.

To consider differences in performance on each type of question, scores on the separate question content types were analyzed in a multivariate analysis of covariance, MANCOVA, with pre-instruction magnitude comparison scores as a covariate. Using a Bonferroni-Holm correction for multiple comparisons, condition had a significant effect on the “how many” questions, $F(1, 24) = 6.83$, $p = 0.015$, $\eta_p^2 = 0.22$, with participants in the pre-made generic condition scoring higher than those in the student-made art condition. Scores on the magnitude comparison questions were also higher in the pre-made generic condition than in the student-made art condition, $F(1, 24) = 4.80$, $p = 0.038$, $\eta_p^2 = 0.17$. No significant difference between conditions was found for scores on the labeling questions, $F(1, 24) = 0.31$, $p = 0.59$.

In both conditions, scores on the labeling questions were well above a chance score of 25% correct, $t(14) = 11.1$, $p < 0.001$, $d = 2.85$, $t(11) = 12.8$, $p < 0.001$, $d = 3.71$ for the student-made art and pre-made generic conditions respectively. Scores on the magnitude comparison questions were also well above a chance of 50% correct in both conditions, $t(14) = 2.96$, $p < 0.02$, $d = 0.76$, $t(11) = 6.14$, $p < 0.001$, $d = 1.77$ for the student-made art and pre-made generic conditions respectively. Additionally, the magnitude comparison questions given during instruction were the same as those of the pre-instruction test. In terms of improvement, participants in the pre-made generic condition had significant gains of 22.2%, $t(11) = 4.30$, $p < 0.002$, $d = 1.24$, while gains for participants in the student-made art condition (8.89%) were not reliably different from zero, $t(14) = 1.22$, $p = 0.24$, $d = 0.32$.

Day 2: Delayed test

Total delayed test scores were also different between conditions. Participants in the pre-made generic condition scored higher than those in the student-made art conditions, ANCOVA, $F(1, 24) = 5.27$, $p < 0.04$, $\eta_p^2 = 0.18$. Pre-instruction magnitude comparison scores also accounted for part of the delayed test score, $F(1, 24) = 5.85$, $p < 0.03$, $\eta_p^2 = 0.20$.

A multivariate analysis, MANCOVA, with pre-instruction magnitude comparison scores as a covariate and a Holm-Bonferroni correction, shows a significant effect of condition on the labeling questions, $F(1, 24) = 9.55$, $p < 0.01$, $\eta_p^2 = 0.29$. Participants in the pre-made generic condition scored higher than those in the student-made art condition. The differences in scores on the estimation and magnitude comparison questions were not significant, $F(1, 24) = 1.93$, $p = 0.18$ and $F(1, 24) = 1.86$, $p = 0.19$ respectively.

Scores on the fraction labeling questions were above chance (25% correct) in both conditions, $t(14) = 6.83$, $p < 0.001$, $d = 1.76$ and $t(11) = 36.4$, $p < 0.001$, $d = 10.5$ for the student-made art and pre-made generic conditions respectively. Scores on the estimation questions were also above chance (25% correct) in both conditions, $t(14) = 4.40$, $p < 0.002$, $d = 1.13$ and $t(11) = 6.31$, $p < 0.001$, $d = 1.82$ for the student-made art and pre-made generic conditions respectively. For the magnitude comparison questions, scores were significantly above chance in the pre-made generic condition, $t(11) = 2.99$, $p < 0.02$, $d = 0.86$, but were not necessarily above chance in the student-made art condition, $t(14) = 1.85$, $p = .085$, $d = 0.48$.

Discussion

Overall, the results of Experiment 1 suggest that the math-and-art activity hindered learning and transfer of fraction knowledge in comparison to the activity with the simple, generic material. Participants who used the simple, pre-made material scored higher than those who used the rich, student-made material on both the test of learning given during instruction and the delayed test given one week after instruction. More specifically, the rich student-made material had different effects on fraction knowledge depending on the level of complexity. For simpler aspects of fraction knowledge, namely basic fraction labeling, the student-made material did not hinder initial learning in comparison to the generic pre-made material; day 1 fraction labeling scores were equivalent in both conditions. However, the student-made material did hinder transfer one week later. Participants in the pre-made generic condition ably transferred fraction-labeling knowledge to novel material on day 2, while participants in the student-made art condition were markedly less able to do so. For more complex aspects of fraction knowledge related to magnitude and estimation, the student-made material hindered initial learning in comparison to the simple, pre-made material; day 1 scores on magnitude comparison and "how many" questions were lower in the student-made art condition than in the pre-made generic condition. However, no significant differences between conditions on magnitude and estimation questions were present after a one-week delay.

While the results of Experiment 1 support our hypothesis, Experiment 1 involved a small number of participants, and it is possible that a disproportionate number of participants with high levels of prior fraction knowledge ended up in the pre-made generic condition compared to the student-made art condition. This possible difference in prior knowledge may have led to higher pre-instruction labeling scores in the pre-made generic condition than in the student-made art condition. It is also possible that even though there was an effect of condition on learning scores when controlling for pre-instruction labeling scores, potential differences in prior knowledge could result in higher learning scores and higher delayed transfer scores in the pre-made generic condition than the student-made art condition. The goal of Experiment 2 was to directly test the effect of the perceptually rich, contextualized representation (without the activity of constructing the material) versus the generic representation on spontaneous fraction labeling. Children with some basic knowledge of fractions were asked to write fractions to describe proportions presented in either a perceptually rich, contextualized format (i.e., proportions of pizzas) or a generic format (i.e., proportions of circles). We predicted that the extraneous information in the contextualized material would distract children from the relevant mathematical relation between the numerator and denominator that defines the concept of fraction, while the generic material would allow easier recognition of the relevant relation. Therefore, we expected children to be more accurate writing fractions to describe proportions of generic circles than proportions of contextualized pizzas.

Experiment 2

In this experiment, first-grade students were given an open-ended test of fraction labeling similar to the pre-instruction labeling test of Experiment 1. Because we were interested in the application of prior fractions knowledge and not the learning of fractions, participants were students from classrooms in which there had been some previous instruction on fractions. Unlike Experiment 1, we gave participants no instruction on fractions; participants were simply shown visual proportions and asked to write fractions to describe these proportions.

Method

Participants

Fifty first-grade students (25 boys and 25 girls, M = 7.23 years, SD = 0.31 year) were recruited from three public schools and one private school in the suburbs of a Midwestern city in the United States. Parents gave written consent for their children to participate. The majority of participants were Caucasian from middle-class families. Of twelve classroom teachers, we initially contacted to recruit participants, eight had introduced students to basic fraction notation. Participants were from these eight classrooms in which there had been some exposure to fractions. The experiment was conducted in the last month of the academic year.

Design

There were two between-subject conditions, contextualized-then-generic and generic-then-contextualized, which specified the order in which the questions were presented.

Materials

Participants were given a 24-question, paper and pen test of fraction labeling. Each question presented a proportion and participants' task was to write down the fraction that described the proportion. The questions were broken into four sets of six questions: contextualized labeling, generic labeling, contextualized estimation, and generic estimation. The contextualized questions presented proportions of pizza, and the generic questions presented proportions of circles. The labeling questions presented pizzas or circles that were divided into equally sized pieces with demarcation lines. These questions were in the same format as the questions used for the pre-instruction labeling test in Experiment 1 (see Fig. 1). There were no demarcation lines for the estimation questions.

Participants in both conditions answered the same questions, but in different orders. In the contextualized-then-generic condition, the question sets were presented in the following order: contextualized labeling, generic labeling, contextualized estimation, and generic estimation. In the generic-then-contextualized condition, the question sets were presented in this order: generic labeling, contextualized labeling, generic estimation, and contextualized estimation. All questions were presented in two paper booklets. The first booklet presented only the first set of questions (i.e., contextualized labeling for the contextualized-then-generic condition and generic labeling for the generic-then-contextualized condition). The remaining three sets of questions were presented in the second booklet.

Procedure

The experiment was conducted in the participants' classrooms. Within each classroom, participants were randomly divided into two groups to form the two between-subjects conditions and then seated at desks. The experimenter passed out the first set of paper booklets. The experimenter instructed participants in the contextualized-then-generic condition to write down the fraction that describes the proportion of pizza that is left for each of the questions. In the generic-then-contextualized condition, participants were instructed to write down the fraction that describes the proportion of the circle that is grey for each of the questions. Participants completed the questions in the booklet at their own paces. When everyone had answered all the questions, the experimenter collected the first set of booklets and then passed out the second set of booklets and gave instructions to write the fraction that describes the appropriate proportion, namely the proportion of pizza left or the proportion of the circle that is grey. Participants answered those questions at their own paces. When participants completed all the questions, the experimenter collected the booklets.

Results

Table 5 presents mean accuracy, split by condition and question type. Note that accuracy in this experiment is higher than that observed on the pre-instruction test of Experiment 1; this is because all participants in this experiment, unlike Experiment 1, were from classrooms that had some previous instruction on fractions. We made several between-condition comparisons of participants' performance. First, we compared performance on the first set of questions to consider the effect of format on spontaneous responding. Participants in the generic-then-contextualized condition scored significantly higher than those in the contextualized-then-generic condition on their initial question set, 78% versus 40% respectively (see Table 5), independent-samples t test, $t(48) = 3.39$, $p < 0.002$, $d = 0.959$. This result replicated the difference in pre-instruction test scores of Experiment 1. In both experiments, participants who were presented with the generic questions were markedly more accurate than participants who were presented with the contextualized questions. This finding suggests that the generic format was more likely to prompt spontaneous correct responses to fraction labeling questions than the contextualized format.

Table 5 Mean accuracy (percent correct) in Experiment 2, split by condition and question type. Standard deviations appear in parentheses

Question type	Condition	
	Contextualized-then-generic (n = 25)	Generic-then-contextualized (n = 25)
Labeling		
Contextual	40.0 (42.2)	75.3 (38.5)[a]
Generic	51.3 (44.3)	78.0 (36.8)[b]
Estimation		
Contextual	46.0 (42.3)	74.0 (37.6)[b]
Generic	46.0 (41.2)	71.3 (37.7)[b]

[a]Significant differences between conditions at $p < 0.01$
[b]Significant differences between conditions at $p < 0.05$

In addition to comparing performance on the initial question set, we also compared performance on contextualized labeling questions and generic labeling questions between conditions to investigate the effect of the order in which questions were answered. Scores were submitted to a repeated measures ANOVA with condition as a between-subject factor and question type as a within-subject factor. Results reveal a significant effect of condition, $F(1, 48) = 7.70$, $p < 0.01$, $\eta_p^2 = 0.14$. Participants in the generic-then-contextualized condition scored significantly higher than participants in the contextualized-then-generic condition on both the contextualized questions and the generic questions (see Table 5), $t(48) = 3.09$, $p < 0.01$, $d = 0.87$ and $t(48) = 2.31$, $p < 0.03$, $d = 0.65$ respectively. Question type also had a significant effect on accuracy, $F(1, 48) = 6.96$, $p < 0.02$, $\eta_p^2 = 0.13$. Participants were more accurate on generic questions than on contextualized questions. The interaction between condition and question type was not significant, $F(1, 48) = 2.67$, $p = 0.109$.

To further examine participants' responses, we categorized participants by their predominant type of response on the initial question set and separately on the second question set. If greater than 50% of a participant's responses on a question set were correct, then he or she was categorized as *correct*. Otherwise, a participant's responses were placed into one of four categories: (1) *numerator:* responses equal to the numerator of the correct fraction (e.g., response of 1 when 1/4 is correct), (2) *opposite*: responses equal to the fraction that describes the opposite proportion (e.g., response of 3/4 when 1/4 is correct) or responses that were reciprocals of the correct response (e.g., response of 4/1 when 1/4 is correct), (3) *ratio:* responses equal to the ratio of the numerator and the difference of the denominator and the numerator (e.g., response of 1/3 when 1/4 is correct), and (4) *other*. Responses in the "other" category included seemingly random numbers or equations involving addition or subtraction (e.g., 1 + 3 = 4). A participant's responses were placed into one of these four categories if at least 50% of the responses met the category definition.

Table 6 presents the percentage of participants whose responses fell into each category for both the initial question sets (shown in the table under 1st) and second question sets (shown in the table under 2nd). The generic-then-contextualized condition had more correct responders and fewer numerator responders than the contextualized-then-generic condition. The distribution of participant responses on the initial sets differed by condition. An asymmetric log-linear analysis with response category as the dependent variable found condition to be a significant factor, $\chi^2(1, N = 50) > 11.6$, $p < 0.03$. The differences were significant even if we include participants in the "opposite" response category (who arguably had some knowledge of fractions, but did not respond accurately) in the "correct" category, $\chi^2(1, N = 50) > 8.78$, $p < 0.04$. Similar analysis for responses on the second set of questions found no significant differences between conditions, $\chi^2(1, N = 50) = 7.02$, $p = 0.135$.

In addition to differences between conditions on the fraction labeling questions, there were striking differences on the estimation questions (see Table 5), with participants in the generic-then-contextualized condition scoring higher than those in the contextualized-then-generic condition. A repeated measures ANOVA with condition as a between-subjects variable and question type as a within-subject variable revealed a significant effect of condition, $F(1, 48) = 5.88$, $p < 0.02$, $\eta_p^2 = 0.11$. Participants in the generic-then-contextualized condition scored higher than participants in the contextualized-then-generic condition on both the contextualized questions and the generic questions, $t(48) = 2.47$, $p < 0.02$, $d = 0.70$ and $t(48) = 2.27$, $p < 0.03$, $d = 0.64$ respectively (see Table 5). No significant differences in accuracy were found between question types, $F(1, 48) = 0.324$, $p = 0.572$. There was no significant interaction between condition and question type, $F(1, 48) = 0.324$, $p = 0.572$.

Discussion

The results of Experiment 2 demonstrate that participants were more likely to write correct fractions when shown proportions of circles than when shown proportions of pizzas. The percentages in Table 6 also show that participants in the contextualized-then-generic were five times more likely than those in the generic-then-contextualized condition to focus only on the number of slices present/shaded (i.e., the numerator) instead of the proportion of those slices out of the entire group (see Table 6 under numerator).

Moreover, exposure to the initial question format influenced participants' performance on subsequent questions; those who initially answered the generic circle questions were more likely to give correct responses to subsequent contextualized pizza questions than participants who initially answered the pizza questions. In addition, participants who initially answered the generic questions were more accurate on the estimation questions for which they needed to determine a fraction that would best describe a proportion presented without demarcation lines that precisely indicate the proportion. For participants who first answered the contextualized questions, the exposure to these questions appears to have hindered their accuracy not only on those questions but also on subsequent generic labeling questions and estimation questions.

General discussion

Teaching young children mathematics can often be challenging in part because effective instruction needs to communicate the relational structure of the mathematics while maintaining children's attention on the learning task. One approach to this challenge is to engage students in instructional activities with physical material. The results of our survey indicate that many teachers in the United States believe that incorporating such activities into math lessons can increase students' engagement and math learning outcomes. The survey results also indicate that many

Table 6 Percentage of participants in Experiment 2 in each response category on the initial and second question sets

		Percentage of participants in response categories on the first and second fraction labeling question sets									
		Correct		Numerator		Opposite/reciprocal		Ratio		Other	
Question set:		1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd
Condition:	*n*										
Context-generic	25	36	48	20	20	12	8	8	8	24	16
Generic-context	25	80	76	4	4	4	8	0	0	12	12

teachers regularly use instructional activities involving a variety of different materials. These include activities that integrate mathematics with art by having students make representations that are then used for instruction. Survey respondents indicated that their choices of activities and materials are more often inspired by informal online resources, such as Pinterest and YouTube, than research-based resources or their school-designated curriculum.

The survey results also show that most teachers use the representations that are explicitly recommended in the standard curriculum (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010); these are number lines, area models, and strip diagrams. However, teachers stated that they are more likely to use pizzas and other contextualized representations of fractions in the future than to use the number line. They rated the number line as the least effective representation of those presented in the survey. This is surprising because the number line is a standard mathematical representation and instruction with the number line can improve whole number magnitude and arithmetic knowledge (e.g., Booth & Siegler, 2008) as well as basic fraction knowledge (Fazio, Kennedy, & Siegler, 2016; Hamdan & Gunderson, 2017). Knowledge of the number line has been shown to be predictive of mathematics achievement (Booth & Siegler, 2006; Siegler & Booth, 2004; De Smedt, Verschaffel, & Ghesquiere, 2009; Geary, 2011; Sasanguie, Van den Bussche,& Reynvoet, 2012). First graders' accuracy on a number line task predicts their mathematics achievement through 5th grade, even after controlling for intelligence, working memory, processing speed and other early numerical skills (Geary, 2011). Given these findings, it is concerning that some teachers may be less likely to use the number line for instruction than many non-standard, contextualized representations.

While the use of instructional activities involving contextualized representations, such as student-made paper pizzas, appears to be widespread, there has been no clear evidence that such activities effectively or efficiently promote mathematics learning and transfer. Intuitively, it may seem that such activities can promote learning because they are engaging. However, with respect to effective instructional material, the intuitions of even experienced teachers do not always align with research findings (Dorward, 2002).

Experiment 1 investigated the effectiveness of such an activity by examining learning and transfer from colorful, contextualized student-made material versus simple generic pre-made material. Overall, there was an advantage for the pre-made material over the student-made material for both initial learning and subsequent transfer; this result contradicts responses to our survey in which teachers rated the pizza activity as slightly more effective than the circle activity. Specifically, for the simpler concept of basic fraction labeling, there was no disadvantage with respect to initial learning for the student-made material compared to the pre-made material, but there was a disadvantage for the student-made material regarding transfer to novel representations after one week. This result may suggest that for students who learned with the student-made material, their internal representation of the newly acquired fraction knowledge may be tightly bound to the extraneous conceptual and perceptual information of the colorful contextual material. This would allow them to recognize fractions in the context of the learning material, but make it difficult to recognize in the absence of the learning material. The generic pre-made material had less extraneous information to overwhelm the internal representation, making recognition of fractions in novel contexts easier. For the more complex concepts involving fraction magnitudes, there was an advantage for the pre-made generic material over the student-made material during initial learning, but this advantage attenuated for delayed transfer. Magnitude judgments are relationally more complex than simple fraction labeling because the learner needs to reason about two fractions simultaneously. The present results suggest that the effect of the colorful, contextualized student-made material may interact with the level of complexity of the to-be-learned concept. Under conditions of increased relational complexity, children may not be able to inhibit the irrelevant perceptual and contextual information of the student-made representations thus increasing the difficulty of the learning task. This possibility is supported by research demonstrating that developmental improvements in children's relational reasoning can be modeled through a computer simulation by increasing inhibition levels to simulate maturation of inhibitory control (Morrison, Doumas, & Richland, 2011); increases in inhibition levels led to increased relational responses.

The results of Experiment 2 support the hypothesis that the perceptually rich, contextualized representations themselves (in absence of constructing them as part of a math-and-art activity) can hinder children's ability to spontaneously recognize and apply mathematical knowledge in comparison to decontextualized generic representations. Children who had some previous exposure to fractions in school were markedly more likely to write correct fractions to describe generic proportions than to describe perceptually rich, contextualized proportions. Moreover, children who initially labeled generic proportions were subsequently more accurate labeling the contextualized proportions than children who labeled the contextualized proportions first. Participants in the generic-then-contextualized condition were also more accurate using fractions to label estimated proportions (both generic and contextualized) than participants in the contextualized-then-generic condition. These findings suggest that the simple, generic format facilitated the recognition of the relation that defines a fraction (i.e., *a/b*, the relation between the numerator and the denominator), while the colorful, contextualized format was less likely to do so. This possibility is supported by the analysis of specific participant responses. Participants in the contextualized-then-generic condition were five times more likely than those in the generic-then-contextualized condition to respond by providing the numerator of a fraction describing the proportion as opposed to providing an entire fraction (i.e., 20% versus 4%). Participants who responded with the numerator may have attended only to the number of slices of pizza remaining or colored sectors of the circle and not to the total number of pieces. The tendency to focus only on the number of slices remaining may have been particularly strong for the contextualized questions because the slices of pizza themselves are perceptually very salient. The salience of the pizza slices may have captured participants' attention, not allowing it to be allocated to all relevant pieces of information, namely the number of elements in the specified subset (e.g., number of slices of pizza remaining), the number of elements in the entire set (e.g., total number of slices remaining or missing), as well as the relation between these numbers. There were also more participants in the contextualized-then-generic condition than in the generic-then-contextualized condition who responded incorrectly by answering with the ratio, the reciprocal fraction, or the fraction that described the opposite proportion. These participants did attend to both the number of elements in the subset and the total number of elements, yet they did not respond with the correct fractions. Therefore, it appears that the perceptually rich, contextualized format distracted some children from attending to the entire number of elements in the set. For other children who did attend to the entire set, the contextualized format was less likely to prompt accurate fraction labeling in comparison to a generic format.

The results of Experiment 2 also demonstrate that the hindering effects of short exposure on perceptually rich, contextualized representations can last beyond the time when the material is present. Accuracy of participants in the contextualized-then-generic condition was lower than that of participants in the generic-then-contextualized condition not only on the initial question set but also on subsequent generic questions. At the same time, initial exposure to the generic representation in the generic-then-contextualized condition also produced a lasting effect by facilitating the recognition of the relevant relations and accurate fraction labeling on subsequent contextualized questions.

Taken together, the results of Experiments 1 and 2 suggest that the colorful, contextualized representations of proportion hindered the spontaneous ability to recognize the mathematical relations that define the concept of fraction and the ability to learn and transfer fraction knowledge. While the material used in the two conditions of Experiment 1 differed on three dimensions: student-made versus pre-made, perceptually rich versus perceptually sparse, and contextualized versus decontextualized, the results of Experiment 2 suggest that the appearance of the material itself, in absence of making it, hindered students' ability to recognize fractions. As mentioned earlier, it is also possible that the act of constructing the material prior to instruction contributed to lowering levels of learning and transfer in the student-made material condition of Experiment 1. Making the material increased students' non-symbolic experience with the material; previous studies have shown that increasing non-symbolic use of an object decreases the likelihood that children will use the object as a symbol for something else (DeLoache, 1995, 2000; Uttal, et al., 1995). Therefore, the act of making the material may make it more difficult for the student to see the material as a symbol for the mathematics it is intended to represent.

There are a few limitations of this study. First, the teacher survey was administered online. As such, the group of respondents may have included teachers who are comfortable with computers and technology, but omitted some teachers who are less comfortable with these. However, given that teachers have active email addresses and many schools have student grade information available to parents online, we expect that most teachers are relatively comfortable using computers and use them for some aspects of their work. Therefore, in this sense, we expect that the sample is relatively representative of the population of practicing elementary school teachers in the United States. In addition, the survey results include only responses of teachers who chose to take the survey. Teachers who were willing to take the time to complete the survey may perhaps have different beliefs and practices than teachers who did not respond. Respondents may be more engaged in their teaching, more receptive to educational research, more receptive to a variety of instructional approaches, or different in some other way from teachers who chose not to respond. If this is the case, then the survey findings may not generalize to all teachers.

A second limitation is that the sample size of Experiment 1 was relatively small. While the results of Experiment 2 support those of Experiment 1, future studies can extend the research by examining the effect of student-made representations on the acquisition of different mathematical concepts and at different stages of development. The advantage of instruction with generic pre-made material over instruction with the colorful, contextualized student-made material would likely generalize to the acquisition of other mathematical concepts because mathematical concepts are defined by relations and are independent of superficial features of specific contexts (see Kaminski & Sloutsky, 2011 and Kaminski, et al., 2013 for discussion). Therefore, integrating mathematics instruction with other activities and perceptually rich, contextualized material often adds extraneous information that can potentially distract the learner from the relevant relations. The likelihood and degree of negative effects on learning and transfer due to the extraneous information would depend on the relational complexity of the concept being learned and the level of development of the learner. Lower relational complexity and/or higher level of a student's attentional focus would likely minimize such negative effects. Defining the precise level of relational complexity at which differences arise requires additional research, but would likely depend on children's level of development and prior mathematical knowledge.

The present research did not separate the effects of the perceptual richness from those of contextual familiarity. While previous research has shown that unfamiliar, irrelevant perceptual richness can hinder both learning and transfer even in adults (Sloutsky, et al., 2005), irrelevant perceptual richness may have advantages for young children's ability to apply previously acquired mathematical knowledge. Pre-schoolers were more accurate on a counting task involving perceptually rich, unfamiliar objects than on the same task involving either perceptually rich, familiar objects or perceptually sparse objects (Petersen & McNeil, 2013). This evidence suggests that the hindering effects of the perceptually rich, contextualized material (i.e., pizzas) on children's ability to label proportions with fractions in Experiment 2 may be due primarily to the familiar contextualization and not necessarily the perceptual richness itself. In other words, viewing proportions instantiated with familiar material may activate extraneous information (e.g., "I like pizza", "it has pepperoni", etc.) that diverts attention from the relational structure of the mathematics. In the context of known mathematical concepts, irrelevant perceptual richness, like that used in the Petersen and McNeil study, may activate little conceptual knowledge and hence have little power to divert attention from the relevant relations.

Experiment 2 involved participants who had some previous instruction on fractions, but the extent of instruction and the format of instructional material they had are not known. Therefore, we do not know the extent to which participants may have previously seen fraction representations similar to those used in Experiment 2. Simple area models, such as circles, and contextualized models, such as pizzas, appear in many textbooks and instructional materials (e.g., Beckman, 2018) and most of the survey respondents indicated that they have used both circles and pizzas as fraction representations. Therefore, we expect that both the generic and contextualized questions are similar to common instructional material. However, it is possible that participants may have had more exposure to simple representations similar to circles than to contextualized representations similar to pizza. Area models are explicitly recommended in curriculum standards (e.g., National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), and our survey results also indicate that more teachers have used the circles than pizzas to represent fractions. If the generic questions of Experiment 2 were more similar in format to participants' previous experiences with fractions than were the contextualized questions, then higher similarity between learning and transfer contexts could in part explain the higher accuracy on the generic questions than on the contextualized questions. If this is the case, it demonstrates that participants' fraction learning from simple representations, similar to circles, accurately transferred to the generic questions as well as the contextualized questions. However, this possibility does not explain the effects of question order on accuracy and why initially viewing the contextualized questions hindered accuracy on all of the subsequent questions. As we have suggested, contextualized representations may be difficult to view as representations of mathematics because they communicate additional, non-essential information that simple generic representations do not. In terms of the material of Experiment 2, young students may see a proportion of a circle primarily as something that stands for a fraction. Proportions of circles appear to have activated previous fraction knowledge that allowed participants to correctly answer generic questions as well as subsequent contextualized questions. At the same time, young students may see a proportion of a pizza primarily as something you can eat. Once this nonmathematical interpretation of the material is activated, it may be difficult for children to spontaneously ignore, making it difficult to correctly answer both the contextualized questions and the subsequent generic questions.

It is important to contrast the present study with other research and educational initiatives that explore students' mathematical reasoning in the course of making things. For example, the Recrafting Mathematics Education project (e.g., Gresalfi & Chapman, 2017) examines students' reasoning, problem-solving, and design through knitting and making other textile crafts. The researchers suggest that making crafts from textiles may be a one way of engaging some students, particularly female students, in mathematical reasoning.

Findings suggest students can successfully problem solve and notice patterns in the context of such craft making (Gresalfi & Chapman, 2017). Other research has examined how aspects of STEM (science, technology, engineering, and mathematics) education might be integrated with philosophies of the maker culture. This culture encourages learning through communities that use a "do-it-yourself", less formal approach to learning to make things, ranging from textile and wooden crafts to electronics and computer code. For example, maker environments have been successful at teaching aspects of basic electronic circuitry through activities in which students use conductive thread to sew circuits into fabrics connecting sources and loads, such as LED lights (Peppler, & Glosson, 2013; Peppler, Gresalfi, Tekinbas, & Santo, 2014; Peppler, Halverson, & Kafai, 2016). Maker-environments can generate student interest and allow for tinkering and trial and error, which can promote learning in the given domain.

These educational initiatives demonstrate that craft-oriented and maker-oriented programs have benefits; they can promote student interest in applications of science and mathematics. They can give students opportunities to practice applying basic mathematics and/or to learn the basics of scientific concepts. However, neither has been shown to promote learning of a new mathematical concept and transfer of this knowledge to standard symbolic mathematics and new domains. The research on knitting and textile craft activities indicates that students can recognize and apply mathematics of counting, measurement, and arithmetic to problem solve in these craft domains, particularly when they are already familiar with the domain (Gresalfi & Chapman, 2017). This is evidence that students can recognize and apply previously learned mathematics, not acquire new mathematics. Successful learning of basic circuitry from maker activities is evidence that students can learn the basics of a scientific concept through hands-on, tinkering, making experiences. However, it is unclear whether successful learning in these concrete contexts translates to knowledge of formal principles, such as Ohm's Law of electrical current, or the ability to apply what was learned to new contexts. In contrast, the goal of Experiment 1 was to examine learning of new mathematical information through an activity in which students made material to represent the to-be-learned mathematics and transfer of this information to standard symbolic mathematics and novel questions.

Conclusions

The results of the present study underscore the importance of researching the effectiveness of instructional activities that teachers actually use in classrooms, many of which are not explicitly part of the standard curriculum but are inspired by informal sources. With respect to hands-on activities with physical material, previous research has shown that some can be effective (e.g., Siegler & Ramani, 2009; Tsang, et al., 2015), but that is not always the case (e.g., Ball, 1992). The effectiveness of activities with manipulatives depends on both the manner in which students interact with the material (Tsang, et al., 2015) and the material itself. Manipulating physical representations of mathematical concepts can provide students with perceptual information that correlates with the mathematical structure and may facilitate learning, but both the material and the student interactions with it need to promote recognition of the relevant mathematical structure. Our findings suggest that perceptually rich, contextualized material, including those that are made by the student, can make this recognition difficult in comparison to simple generic material. Previous research has shown that fading perceptually rich material into perceptually sparse, more generic material can improve learning and transfer (Fyfe, McNeil, Son, & Goldstone, 2014; Goldstone & Son, 2005; McNeil & Fyfe, 2012). However, as Tsang et al. (2015) suggest, when learning from interaction with physical material, learners can build an appropriate mental model of the mathematics, but it is likely that residual traces of the physical material and actions on the material remain (see Kaminski & Sloutsky, 2011 for a related discussion). Making elaborate, colorful, contextualized representations would likely increase the extraneous, potentially irrelevant information that is stored as part of the mental representation of the mathematics, even if fading or other measures to connect the student-made material to more generic representations were employed.

We are not suggesting that activities that integrate mathematics and arts, crafts, or other contextualized settings should never be used in the classroom. However, the present results do suggest that for initial instruction on a new or recently introduced elementary mathematical concept, colorful, contextualized student-made material should be avoided or used with caution, particularly with young children.

Abbreviations

NCTM: National Council of Teachers of Mathematics; STEM: Science Technology Engineering and Mathematics

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Authors' contributions

The first author conceptualized the work, oversaw the research, and wrote the paper. The second author contributed to the design of the research and the writing of the paper. All authors read and approved the final manuscript.

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Competing interests
The authors that they have no competing interests.

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